The optimization research of the multi-response problems based on the SUR

Haitao Su*, Hong Mei Yao and Hui Zeng
School of Economics and Management, Nanchang University, Jiangxi Province, China

Abstract: In the optimization design of products and processes in the biological medicine, we need to consider multiple characteristics of quality simultaneously, namely multi-response problems, multi-response optimization design can improve the quality of the products effectively, and realize enormous economic benefits and so multi-response optimization design is showing a more and more important role in continuous quality improvement activities. But usually there is no specific set of input variables to make all the response variables be optimal, and the traditional multi-response surface method cannot solve the correlation problem between multi-responses and regression model problem effectively. Because we can make a better fitting model and solve the problem of the correlation between the response variables at the same time with SUR method, this thesis uses the SUR method to model the relationship between each response and control variables, and makes predictions; confirms the satisfaction function of each response and the overall satisfaction function; combines with practical problems of a company in biological medicine field named SX to conduct empirical research, this thesis confirms the optimal factor level combination with the overall satisfaction function in the end, thus solves the multi-response optimization problems.

Keywords: SUR, multi-response problems, optimization.

INTRODUCTION

In the actual design of product and process optimization, the issue of multiple quality indexes often involves the problem with multiple responses. The multiple response optimization in enterprise quality improvement activity plays an important role.

For the research of multi-response optimization problems, most scholars mainly focus on the optimization method, such as fuzzy logic (Lin et al., 2000 and Lin et al., 2002), four steps of the method based on neural network and principal component analysis (PCA), combined with neural network and genetic algorithm method, optimization algorithm based on principal component analysis (PCA), TOPSIS method as well as other methods (Chi-Ming 2001 and Patricia et al., 2012). There were also some scholars who mainly aimed at the international traditional multiple response optimization method to carry on the analysis on the comparative study and some of these methods have been improved. Zheng He and Yuxuan Zhang found it hard to consider the process of economy and each response variance covariance matrix of the structure when they had analysed the markov distance method and the quality loss function method, so they raised an improved markov distance function method to solve multiple response optimization problems (He et al., 2003).

In recent years, on the multiple response optimization problems, the scholars mainly study the problem how to improve the regression model and better the control variables as well as the responses in multi-response problem. Some scholars put forward: Unrelated regression method (Harry et al., 2004 and John et al., 2009), the method of fuzzy theory (Li et al., 2012), bayes method (Guillermo et al., 2002), neural network method (Parikshit et al., 2007 and Wang et al., 2012), semiparametric regression (Terrence et al., 2005) and so on, and use these methods to solve the problem more effectively. Szu Hui NG studied lots of current optimization methods of multi-response and found many of them haven’t considered the correlation between the responses, the uncertainty of response model, the uncertainty of model parameters estimation. He therefore believes if one can’t deal with these uncertainties, one will cause quality prediction misleading easily. In this article, the authors put the bayesian decision theory into the model of response system and the optimization process, this method can fully solve the uncertainty problem as we said there-in-before (Suz 2010). There are also many scholars who put forward different methods, for example, principal component analysis (pca), Partial least-squares regression model, etc. hoping to get better optimization results. Xiao-Fang Zhong, Zhijun Han found the Multiple index robustness of the design is not enough by studing the method of SN, and put forward a kind of optimization method based on taguchi quality loss function and principal component analysis (Zhong et al., 2003). Jianjun Wang, Yi-Zhong Ma etc used multivariate partial least squares (PLS) regression model to solve the above problem (Wang et al., 2011).

*Corresponding author: e-mail: yhm965908425@qq.com
In summary, the previous studies on response problem, mainly focus on the multi-response optimization problem and the comparison, analysis and improvement of the classic methods, the optimization methods are: PCA, WPC, PCA-TOPSIS, PCA-DEA, GRA, ANN-PCA, VIKOR and so on. At last, the problem of the relationship between multi-response optimization in response, the important between the responses and model of response and control variables, are still the hot and the difficulty spot in the multi-response optimization researching. For the SUR (seems unrelated regression model) can model as well as fully consider the correlation between response, this paper tries to put forward multiple response optimization based on SUR, and then made predictions. It is hoped that the model can fit well after thorough and careful reflection on the correlation between the responses by using this method, so that it can realize the optimization of multi-response problems.

Multiple response optimization model based on SUR
In this section, in order to overcome the defect of the model and the relativity problem between the multiple responses while using the traditional response of multiple response surface method, we built a multi response optimization model based on SUR, the ideas of optimization modeling as follows:

(1) Using the SUR to build the regression model between multiple response and control variables;

(2) Putting the model into the satisfaction function in determining satisfaction function of each response, based on the obtained model between each response and control variables, so that to determine the overall satisfaction function;

(3) The optimal factor level combination is determined according to the overall satisfaction function.

To establish the regression model
In this section, in order to overcome the defect of the model and the relativity problem between the multiple responses while using the traditional response of multiple response surface method, we built a multiple response optimization model based on SUR, the ideas of optimization modeling as follows:

The Seemingly Unrelated Regression model raised by Zellner is a method of panel data analysis. In the model of SUR, the equation of the disturbance term is independent on the obtained model between each response and control variables, are still the hot and the difficulty spot in the multi-response optimization researching. For the SUR (seems unrelated regression model) can model as well as fully consider the correlation between response, this paper tries to put forward multiple response optimization based on SUR, and then made predictions. It is hoped that the model can fit well after thorough and careful reflection on the correlation between the responses by using this method, so that it can realize the optimization of multi-response problems.

The optimization research of the multi-response problems based on the SUR

\[ \Lambda = \beta X' \beta \Lambda \beta + \varepsilon \Lambda \beta \]

\[ Y = X \beta + \varepsilon \]

\[ Y = (y_1, y_2, \Lambda, y_i) \Lambda \Lambda X_1, X_2 \Lambda \Lambda X_j \]

\[ \beta = \beta_1, \beta_2, \Lambda, \beta_j \]

\[ \varepsilon = \varepsilon_1, \varepsilon_2, \Lambda, \varepsilon_j \]

respectively represent the response variables and control variables.

In the model of SUR, the errors \( \varepsilon_i \) meets:

\[ E (\varepsilon_i) = 0 \]

\[ Var (\varepsilon_i) = \sigma_i I \]

\[ Cov (\varepsilon_i, \varepsilon_j) = \sigma_{ij} I \]

So, we can get the variance covariance of error \( \varepsilon_i \) which can be expressed as matrix:

\[ V = \Sigma \otimes I \]

\[ \Sigma = \begin{pmatrix}
\sigma_1^2 & \sigma_{12} & \Lambda & \sigma_{1n} \\
\sigma_{21} & \sigma_2^2 & \Lambda & \sigma_{2n} \\
M & M & O & M \\
\sigma_{n1} & \sigma_{n2} & \Lambda & \sigma_n^2 \\
\end{pmatrix} \]

The coefficients of the control variables \( \beta \) in the model reflect the correlation between the response factors, therefore the key point of the SUR regression model in this paper is to ensure \( \beta \), and the Linear unbiased estimator of the \( \beta \) is \( \hat{\beta} \), which can be calculated by the formula as follow:

\[ \hat{\beta} = [X' (\Sigma \otimes I_n) X]^{-1} X' (\Sigma \otimes I_n) y \]

To determine the coefficients of the control variables, we should make sure the estimate of \( \Sigma \) as \( \hat{\Sigma} \) and there is \( \hat{\Sigma} = \hat{\Sigma} \otimes \hat{\Sigma} \), which:

\[ \hat{\Sigma} = y_i [I_n - X_i (X_i' \Lambda^{-1} X_i) [I_n - X_j (X_j' \Lambda^{-1} X_j) \Lambda^{-1} X_j] y_i \]

In the above formula, the \( \hat{\Sigma} \) can be worked out with the residual vector which is obtained by the model of each argument and the least square method to fit the response variable. Therefore, according to the above reasoning and analysis, the steps of the SUR regression model in this paper is as follow:

(1)By using the method of least square (OLS) fitting response between variables and control variables in the model, then get the calculation \( \hat{\Sigma} \) by using the residual vector.

(2) After that, using equation (6) calculate \( \hat{\beta} \), so that to make sure the regression model of SUR.

Ensure the overall satisfaction
Using the SUR model in the above section, We can determine the response regression model between the variables and the control variables: \( y_i = \beta \hat{x}_i \). Then we will put \( y_i \) into the satisfaction function to make out the
satisfaction function of each responses, so that to get the determination of individual response variable satisfaction value.

After determining the single response variable satisfaction value, we should determine the overall satisfaction value in order to obtain the optimal factor level combination, the overall satisfaction value is:

\[ D = \sqrt[4]{d_1d_2d_3K} \]

Then it can determine the optimal factor level combination optimization method based on the SUR.

The analysis of case and the application of model
The case status of the multi-responses problem
The Chinese medical equipment limited-liability company SX, is a national high and new technology enterprise, who specializes in medical equipment research, development, manufacture and sale. It founded in the city of Nan Chang in 1997, took the lead through the CE, CMD quality management system and product certification and the United States FDA510 (K) register in 1999. Now take it as an example, with the multi-response problems of the injection barrel used by syringe injection molding process. In the process of the injection barrel of injection molding, the mold temperature (x1), injection speed (x2) and holding time (x3) etc, the three parameters need to be controlled, so that to make the quality of the injection cylinder to achieve the optimal, which the quality of the injection cylinder can be measured by four quality feature values, they are: sealing separation force (y1), acupuncture (y2), rigid (y3) and withstand pressure (y4).

In order to determine the optimal value of x1, x2 and x3, company SX made the following work:

(1) Determining factor number and its level, and the nature of the response variables. In this case, the factor number and its level, and the requirements of the response variables and properties is as the following table 1.

(2) After determining the number of the factor and its level, and the nature requirements of the response variables, they made test plan. As there were three factors in this case, and each factor had two levels, they put the method Central composite design (CCD) of response surface designs to use, this method covered three part of test points: Angular point, each point coordinates are 1 or -1(which 1 means high level, -1means low level), cent point, each point coordinates are 0, axis point, only one point coordinate is, the others are all 0, in this case there are 6axis point in all, and the is 1.682, the center point repeated three times, so17 times experiment should be carried out in this case.

(3) Play the test according to the above test plan and collect the data of the four response variables, the data as the table 2 shows:

(4) After getting the data on the basis of above test values, the analysts of SX carried on the analysis, and determined the best combination of control variables. They used multi-response surface design, which contains in the Minitab software to solve the problem. There are two tools which can be chosen to optimize the multi-response variable, which are overlapping contour map and response optimizer. In order to get the result more conveniently, the members of SX used the method of response optimizer to determine the best combination level of factors, the results are shown in fig. 1. Which shows that when the best values of control variables were -0.5267 (mold temperature), 1.000 (injection speed), 0.5946 (holding time), the multi-response variables would be optimized, and the optimal values are respectively as: 1.92 (sealing separation force), 19.5139(acupuncture force), 17.2608 (rigid), 49.8592 (withstand pressure force), then the overall satisfaction value would be 0.7144.

![Optimization figure](image)

Fig. 1: The optimized results of injection cylinder multi-response problem

(5) Validation test to test the above result. It means setting the parameters as 84.73(mold temperature), 295(injection speed), 91.89 (holding time). After processing, they got the optimal value of each response variables are respectively as 1.918(sealing separation force), 19.516(acupuncture force), 17.264 (rigid), and 49.851(withstand pressure force). Compared with the optimal values of the theory, the actual values almost did not change, so, the analyst thought that the combination of the control variables was the optimal parameters of setting.

The practice of multi-response problems based on the SUR with the case
For the sake of rigorous, we should analyze the relativity of multi-response among in the process of SX company simple injection. We used the software of SPSS to analyze, the results as the table 3 shows.
The optimization research of the multi-response problems based on the SUR

As the table 3 demonstrates, we can see that there is a strong correlation among the four responses. So, it is needed to consider the correlation of the four responses in the simple multi-response optimization of injection molding process. However, it just can’t simply be solved by the traditional response surface method. Confronted with this kind of problem, if one wants to get the best combination of factor levels in real sense, one must consider the correlation among the response. So it is necessary to make optimization based on SUR.

There were four response variables and three control variables in the case, so the regression model of this case based on the formula (1) can be shown as:

\[ y_{ij} = \beta_0 + \beta_{1j}x_{ij1} + \beta_{2j}x_{ij2} + \beta_{3j}x_{ij3} + \beta_{4j}x_{ij4} + \epsilon_{ij} \]

In the above formula, it means the number of response variable. We should use response surface methodology to get the best combination levels of factor, because there were just three control variables in the case, we just need to fit coincide model, so there is no cubic term in the model as above.

Which: \[ Y = (y_1, y_2, y_3, y_4) \]

\[ X = \text{diag}(x_1, x_2, x_3) \]

\[ X_i = (1, x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5}) \]

\[ \beta = (\beta_1', \beta_2', \beta_3', \beta_4') \]

\[ \beta = (\beta_{0j}, \beta_{1j}, \beta_{2j}, \beta_{3j}, \beta_{4j}, \beta_{5j}) \]

\[ \epsilon = (\epsilon_1', \epsilon_2', \epsilon_3', \epsilon_4') \]

Then, we can get the variance - covariance matrix of error \( E_i \), with the formula (5)

\[
\Sigma = \begin{pmatrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\
\sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} \\
\sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44}
\end{pmatrix}
\]

which,

We use the soft of Mininet to get the OLS regression model of each response, (the operation process was ellipsoid) the four OLS regression model of response variable as follow:

\[ y_1 = 1.8801 - 0.0974x_1 - 0.1029x_1^2 \]

\[ y_2 = 21.2664 + 5.6514x_1 - 1.0184x_1 + 8.1371x_1^2 + 2.5175x_1x_2 \]

\[ y_3 = 179.27 + 742x_1 - 0.002x_1 - 101.7x_1^2 + 4.659x_1^3 + 1.6x_1^2 + 138x_1^2 + 2.13x_1^4 \]

\[ y_4 = 50.7111 + 2.4364x_1 - 3.1515x_1^2 \]

And we can also get the residual vector of the four response variables regression model above, as table 4 shows:

We can get the variance - covariance unbiased estimator of the four residual variables above, the result as follow:

\[
\Sigma = \begin{pmatrix}
0.0017 & -0.0087 & -0.0358 & -0.0085 \\
-0.0087 & 7.5417 & -0.0539 & 2.4999 \\
-0.0358 & -0.0539 & 4.5641 & 4.8344 \\
-0.0085 & 2.4999 & 4.8344 & 14.2182
\end{pmatrix}
\]

After that, we can use the soft of Matlab to fig. out the matrix with formula (7),

\[ \beta_1 = (1.8846, -0.0974, 0, 0, 0) \]

\[ \beta_2 = (22.6488, 5.6148, -0.3411, 0, 2.682, 0, 7.8304, 0) \]

\[ \beta_3 = (18.964, 0.7444, -0.2075, -1.3319, 1.8574, 0.3223, 0.13925) \]

\[ \beta_4 = (51.91, 2.4364, 0, 0, 0, 0, 3.4287, 0) \]

We can ensure the regression model between the response variables and control variables of the case based on the method of SUR:

\[ y_1' = 1.8846 - 0.0974x_1 - 0.1039x_1^2 \]

\[ y_2' = 22.6488 + 5.6148x_1 - 0.3411x_1 + 7.8304x_1^2 + 2.682x_1x_2 \]

\[ y_3' = 89.64 + 7.44x_1 - 1.3311x_1 - 1.3327x_1 - 1.3327x_1 + 1.837x_1 + 1.837x_1 + 1.837x_1 \]

\[ y_4' = 51.91 + 2.4364x_1 - 3.4287x_1^2 \]

According to the actual situation of production site, the biggest value of sealing separation force is 1.92, the smallest value of acupuncture force is 0.1892, the smallest value of rigid is 16, the biggest value of withstand pressure force is 51.56. So, we can fig. out the satisfaction function of each response by using the data and regression model above. The value of each satisfaction function is as follow:

\[ d_i = \frac{y_i - 1.7}{1.92 - 1.7} \]

\[ d_i = \frac{21 - y_i}{21 - 22.6488 - 5.6148x_1 + 0.3411x_1 - 7.8304x_1^2 - 2.682x_1x_2} \]

\[ d_i = \frac{21 - 19.92}{21 - 22.6488 - 5.6148x_1 + 0.3411x_1 - 7.8304x_1^2 - 2.682x_1x_2} \]

\[ d_4 = \frac{y_4 - 4.5}{51.56 - 4.5} \]

As we have obtained the value of each satisfaction function, we need to construct the overall satisfaction function, so as to make sure the best combination level of factors. The formula of overall satisfaction function based on formula (8) as follow:

\[ D = \sqrt{d_1^2, d_2^2, d_3^2, d_4^2} \]

Put the satisfaction function of each response variable into the formula, we can get the mathematical expression of overall satisfaction function. Then the problem of ensuring the best combinations level of factor changed into working-out the value of \( x_1, x_2 \) and \( x_3 \), so that to get the biggest value of overall satisfaction function. We got simultaneous equations by using composite function derivation operation of math as follow:
Putting the satisfaction function formula of each response variable into the simultaneous equations, then fig. out the value:
\[ X^* = (-0.500, 1.000, 0.790) \]
\[ y^* = (1.917, 19.109, 17.014, 49.948) \]
\[ D_{max} = 0.773 \]

It means when the value of control variable were 85 (mold temperature), 295 (injection speed), 95.8 (holding time), the value of overall satisfaction reached the maximum.

### Table 1: Factor number and its level and nature of the response variable

<table>
<thead>
<tr>
<th>Name of factor</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Response variables</th>
<th>Requirements and the nature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mold temperature</td>
<td>80</td>
<td>100</td>
<td>Sealing separation force</td>
<td>≥1.7 Larger the better</td>
</tr>
<tr>
<td>Injection speed</td>
<td>235</td>
<td>295</td>
<td>Acupuncture force</td>
<td>≤21 Smaller the better</td>
</tr>
<tr>
<td>Holding time</td>
<td>60</td>
<td>100</td>
<td>Rigid</td>
<td>≤20 Smaller the better</td>
</tr>
</tbody>
</table>

### Table 2: The test values of each response variables

<table>
<thead>
<tr>
<th>Test number</th>
<th>Control Variables</th>
<th>Response Variables</th>
<th>Sealing Separation force</th>
<th>Acupuncture force</th>
<th>Rigid</th>
<th>Withstand pressure force</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mold temperature</td>
<td>Injection speed</td>
<td>Holding time</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>1.83</td>
<td>29.31</td>
<td>29.50</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>1.73</td>
<td>39.32</td>
<td>19.40</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>1.85</td>
<td>25.16</td>
<td>25.70</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>1.67</td>
<td>40.18</td>
<td>27.10</td>
</tr>
<tr>
<td>5</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>1.86</td>
<td>29.32</td>
<td>21.40</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>1.77</td>
<td>32.20</td>
<td>24.00</td>
</tr>
<tr>
<td>7</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>1.88</td>
<td>22.01</td>
<td>19.60</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1.66</td>
<td>40.02</td>
<td>25.10</td>
</tr>
<tr>
<td>9</td>
<td>-1.682</td>
<td>0</td>
<td>0</td>
<td>1.81</td>
<td>33.00</td>
<td>24.20</td>
</tr>
<tr>
<td>10</td>
<td>1.682</td>
<td>0</td>
<td>0</td>
<td>1.37</td>
<td>51.59</td>
<td>30.60</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>-1.682</td>
<td>0</td>
<td>1.85</td>
<td>20.35</td>
<td>20.90</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>1.682</td>
<td>0</td>
<td>1.92</td>
<td>20.53</td>
<td>18.90</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>0</td>
<td>-1.682</td>
<td>1.88</td>
<td>23.85</td>
<td>23.00</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>0</td>
<td>1.682</td>
<td>1.90</td>
<td>20.16</td>
<td>21.20</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.89</td>
<td>21.72</td>
<td>18.50</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.88</td>
<td>21.21</td>
<td>18.60</td>
</tr>
<tr>
<td>17</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.87</td>
<td>21.55</td>
<td>16.80</td>
</tr>
</tbody>
</table>

### Table 3: The relativity result of multi-response’s simple injection molding process

<table>
<thead>
<tr>
<th>Coefficient of association</th>
<th>Sealing separation force</th>
<th>Acupuncture force</th>
<th>Rigid</th>
<th>Withstand pressure force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sealing separation force</td>
<td>1</td>
<td>-0.934</td>
<td>-0.673</td>
<td>0.024</td>
</tr>
<tr>
<td>Acupuncture force</td>
<td>-0.934</td>
<td>1</td>
<td>0.685</td>
<td>-0.734</td>
</tr>
<tr>
<td>Rigid</td>
<td>-0.673</td>
<td>0.685</td>
<td>1</td>
<td>-0.582</td>
</tr>
<tr>
<td>Withstand pressure force</td>
<td>0.024</td>
<td>-0.734</td>
<td>-0.582</td>
<td>1</td>
</tr>
</tbody>
</table>
In order to show the optimization effect of the multi-response based on SUR, we point it out in two sides of theory and demonstration. Theoretically, we compared the results of traditional method to that of SUR method as shown in table 5.

As the table 5 illustrates, the optimal value of control variable would be more excellent, by using the optimization method based on SUR to model, which made the value of acupuncture force reduce from 19.516 to 19.109, rigid reduce from 17.261 to 17.014, withstand pressure force increase from 49.859 to 49.948, and the overall satisfaction increase from 0.714 to 0.773; from the side of demonstration, we tested the best combinations level of factor of the multi-response problem of injection molding process, and got the test value result of the optimal way (1.92,19.516,17.261,49.859) and found that the optimal value of each response variable was more excellent, compared with the value result of traditional way (1.918, 19.516, 17.264, 49.851). So far, we had testified the significant effect result of the multi-response optimization method (based on SUR) of injection molding process.

**CONCLUSION**

This paper proposed an optimal method of multi-response based on the SUR, after an overall review of relevant studies. Because the model of SUR could do well in modeling as well as deal with the problem of relativity among response variables, it could get the more accurate forecasts value compared with the method of OLS, when there is relativity among the multi-response and the sample is not so small, we got the exact mathematical expressions by using SUR method fitting model between the response variables and control variables; Then we could get the satisfaction function of each response variable by using the method of satisfaction function; At last we used value of overall satisfaction to determine the optimal combination level of factor. This method is a kind of multi-response surface modeling method, which is suitable for the case of few factors. In this article, we used the case of SX Company showing the advantage of the SUR method in the two sides of theory and demonstration, comparing it with the traditional method.

The company who wants to deal with the problem of multi-response optimal can refer to the SUR method proposed in this paper. But there are also drawbacks which limit the usable range of the optimal method, for example, the advantage of the optimal method would not be so effective, if the correlation between the responses was not so strong; for this kind of problem, we had not solved so far because of the limited time and knowledge, we hope the reader can do further research to solve these kinds of problems.

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**Table 4:** The residual vector of the four responses variables regression model

<table>
<thead>
<tr>
<th>Residual vector 1</th>
<th>Residual vector 2</th>
<th>Residual vector 3</th>
<th>Residual vector 4</th>
</tr>
</thead>
<tbody>
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**Table 5:** The Comparative and Analysis of Theory

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<th>Multi-Response problem of injection molding process</th>
<th>Before the optimization (traditional method)</th>
<th>Post optimality (base on the SUR)</th>
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<td>Best combinations level of factor</td>
<td>(84.73,295,91.89)</td>
<td>(85,295,95.8)</td>
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<tr>
<td>Optimal value of control variable</td>
<td>(1.92,19.516,17.261,49.859)</td>
<td>(1.917,19.109,17.014,49.948)</td>
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<td>Value of overall satisfaction</td>
<td>0.714</td>
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REFERENCES


